Unit 1 Answers

1.1 Reciprocals

1 a \( \frac{1}{3} \)
b \( \frac{1}{8} \)
c 8
d \( \frac{1}{9} \)

2 a \( \frac{2}{7} \)
b \( \frac{3}{4} \)
c \( \frac{5}{23} \)
d \( \frac{7}{19} \)

3 b 7
c \( 10\frac{2}{3} \)
d \( 10\frac{1}{2} \)
e \( \frac{9}{20} \)

4 a \( \frac{3}{2} \)
b \( \frac{9}{7} \)
c \( \frac{5}{22} \)
d reciprocal, 1

5 a 5 pF
b \( 1\frac{7}{8} \) pF
c \( \frac{15}{16} \) pF
d \( \frac{12}{19} \) pF

1.2 Indices

1 b \( 2^{-1} \)
c \( 7^{-2} \)
d \( 11^{-2} \)
e \( \frac{1}{27} \)
f \( \frac{1}{32} \)
g \( \frac{1}{8} \)
h \quad 4^{-2} = 2^{-4}

i \quad 25^{-2} = 5^{-4}

2

b \quad 6^{-2}

c \quad 3^{-6}

d \quad 5^{-5}

e \quad 9^{-5}

f \quad 11^{-12}

g \quad 4^{-12}

3

a i \quad 6^{-2}

ii \quad \frac{1}{36}

b i \quad 3^3

ii \quad 27

c i \quad 5^{-3}

ii \quad \frac{1}{125}

4

a \quad \frac{1}{25}

b \quad \frac{25}{64}

c \quad \frac{27}{1000}

5

a \quad \left(\frac{5}{7}\right)^2

b \quad \left(\frac{8}{12}\right)^2

c \quad \left(\frac{9}{10}\right)^2

d \quad \left(\frac{3}{11}\right)^2

e \quad \left(\frac{4}{3}\right)^3

f \quad \left(\frac{1}{5}\right)^3

6

a \quad \frac{1}{8}

b \quad \left(\frac{1}{2}\right)^3

1.3 Standard form

1 \quad 3.5 \times 10^8, 2 \times 10^5, 9.9 \times 10 \text{ and } 4.306 \times 10^{-9}

2

a \quad 5000

b \quad 0.000 37

c \quad 49 000 000

d \quad 0.000 002 09

3 b \quad 4.98 \times 10^2
7.1 \times 10^6  
6.35 \times 10^{-6}  
4 \times 10^{-8}  
4  
a. Mars \ 2.25 \times 10^8, \ our \ moon \ 3.844 \times 10^5, \ Saturn \ 1.3 \times 10^9  
b. Yes  
5  
a. \ 7.8 \times 10^2, \ 8.52 \times 10^2, \ 6.4 \times 10^3, \ 2.1 \times 10^4, \ 3.51 \times 10^4  
b. \ 9.27 \times 10^{-6}, \ 5.31 \times 10^{-5}, \ 1.4 \times 10^{-4}, \ 6.8 \times 10^{-4}, \ 2.67 \times 10^{-3}  
6  
neutron, proton, electron  
7  
a. \ 0.0001 \ m  
ii. \ 0.1 \ mm  
b. \ 0.000 \ 005 \ 1 \ m  
ii. \ 0.0051 \ mm  
1.4 STEM: Calculating with standard form  
1  
b. \ 1.14 \times 10^8  
c. \ 2.52 \times 10^{11}  
d. \ 2.25 \times 10^8 
2  
a. \ 2 \times 10^4  
b. \ 3 \times 10^3  
c. \ 3 \times 10^3  
3  
a. \ 2.5955 \times 10^8  
b. \ 4.7 \times 10^4  
4  
10 \ mm  
5  
a. \ 5.67 \times 10^5  
b. \ 4.77 \times 10^4  
c. \ 4.4 \times 10^{-6}  
d. \ 2.835 \times 10^{-5}  
6  
9.983 \times 10^{-7} \ m  
1.5 Fractional indices  
1  
a. 5 
  b. 12 
  c. 7 
  d. \ \frac{1}{9}  
  e. \ \frac{1}{8}  
  f. \ \frac{4}{11}  
2  
a. 3 
  b. 4 
  c. \ \frac{1}{2}
d $\frac{1}{5}$

3 a $121^{\frac{1}{2}}$  
b $\frac{1}{1000^{\frac{1}{3}}}$  
c $8^{\frac{1}{3}}$  
d $\left(\frac{25}{81}\right)^{\frac{1}{2}}$  
e $3600^{\frac{1}{2}}$  
f $\frac{4}{7}$

4 c $25$  
d $64$  
e $8$  
f $\frac{8}{125}$

5 a $\frac{1}{6}$  
b $\frac{1}{5}$  
c $2\frac{1}{3}$  
d $6\frac{1}{4}$

1.6 Surds

1 c $2\sqrt{7}$  
d $5\sqrt{3}$  
e $3\sqrt{5}$

2 a $5\sqrt{2}$  
b $9\sqrt{5}$  
c $\sqrt{5}$

3 a $2^2 \times 5 \times 7^2$  
b $14\sqrt{5}$

4 $\sqrt{166}$

5 a terminating  
b recurring  
c terminating  
d irrational  
e irrational  
f terminating

6 $\sqrt{169}$ and $\sqrt{\frac{1}{25}}$

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1 Strengthen

Standard form

1  
   a  5200  
   b  38000  
   c  910  
   d  4700000

2  
   a  0.0065  
   b  0.00027  
   c  0.083  
   d  0.0000094

3  
   a  4.9 \times 10^3  
   b  7.3 \times 10^5  
   c  5.1 \times 10^{10}

4  
   a  8.3 \times 10^{-4}  
   b  9.7 \times 10^{-6}  
   c  5 \times 10^{-3}  
   d  4.6 \times 10^{-9}

5  
   a  7.2 \times 10^7  
   b  7.2 \times 10^7  
   c  1.55 \times 10^{10}  
   d  4 \times 10^5  
   e  1.2 \times 10^{-5}  
   f  8 \times 10^3

Reciprocals and indices

6  
   a  \frac{1}{3}  
   b  \frac{4}{5}  
   c  \frac{1}{7}  
   d  \frac{1}{15}  
   e  5

7  
   a  \text{i } \frac{17}{5}  
   \text{ii } \frac{5}{17}  
   b  \text{i } \frac{3}{7}  
   \text{ii } \frac{6}{35}

8  
   a  \frac{1}{64}
b \frac{1}{49}

c \frac{25}{81}

d \frac{1}{16}

e \frac{27}{125}

9 a i \quad 4^{-5}

ii \frac{1}{4^5}

iii \frac{1}{4^5}

b i \frac{1}{5^2}

ii \frac{1}{3^4}

iii \frac{1}{8^7}

iv \frac{1}{7^3}

10 a \quad 5^{-4}

b \quad 4^3

c \quad 7^{-5}

d \quad 3^{-5}

e \quad 9^6

f \quad 6^{-21}

11 a \quad 4

b \quad \frac{3}{8}

c \quad \frac{1}{5}

\quad \frac{1}{10}

12 a \quad 64

b \quad 27

c \quad 25

d \quad \frac{64}{125}

13 a \frac{1}{6}

b \frac{1}{9}

c \frac{1}{2}

\quad \frac{1}{25}
Surds
14 a  $2\sqrt{5}$
   b  $5\sqrt{2}$
   c  $3\sqrt{3}$
   d  $15\sqrt{2}$
15 a rational
   b rational
   c rational
   d rational
   e irrational

1 Extend
1 a 0.0625
   b 0.04167
   c 2.67
   d 20
   e 250
2 a $\frac{1}{125}$ cm$^3$
   b $\frac{27}{64}$ cm$^3$
   c $\frac{4}{5}$ cm
   d $\frac{1}{2}$ cm
3 $3106 \times 10^{-7}, \frac{6.2 \times 10^{-9}}{2 \times 10^{-5}}, 0.000315, 3.17 \times 10^{-4}, 0.31 \times 10^{-3}$
4 a $5 \times 10^{-8}$
   b $2 \times 10^{-7}$
   c $1.25 \times 10^3$
   d $4 \times 10^5$
5 a Switzerland, Austria, Italy, UK, France
   b $1.95 \times 10^6$
   c about 7.5 times
   d $2.076 \times 10^8$
   e about 17.6%
6 a 125
   b $\frac{81}{16}$
   c 81
7 a $(0.2)^3, 5^{-3}$ and $(\frac{1}{5})^3$
   b e.g. $(0.1)^2, 10^{-2}, (\frac{1}{10})^2, 1000^{-\frac{2}{3}}$
KS3 Maths PW Progress Delta 3

8 a 15  
b 6  
c 36  
9 a \(14\sqrt{2}\)  
b \(210\sqrt{7}\)  
10 a \(7\sqrt{5}\)  
b \(5\sqrt{7}\)  
c \(6\sqrt{2} + 2\sqrt{3}\)  
11 height = \(\sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}\)  
area = \(\frac{1}{2} \times 8 \times 4\sqrt{3} = 16\sqrt{3}\)  
12 \(\sqrt[3]{2}, \sqrt[3]{7}, \sqrt[3]{16}\) and \(\sqrt[3]{100}\)

1 Unit test

1 a \(\frac{1}{4}\)  
b \(\frac{27}{1000}\)  
2 a \(\frac{1}{8}\)  
b \(1\frac{2}{3}\)  
c \(\frac{9}{29}\)  
d 10  
3 a \(7.9 \times 10^3\)  
b \(8.13 \times 10^{-6}\)  
4 a \(3 \times 10^{-4}\)  
b \(1.2 \times 10^{-6}\)  
5 a \(5^{-7}\)  
b \(8^{-7}\)  
c \(3^{-8}\)  
d \(6^1\)  
6 a \(4\sqrt{2}\)  
b \(\sqrt{5}\sqrt{7}\)  
c \(6\sqrt{3}\)  
7 a 16  
b \(\frac{1}{27}\)  
c \(\frac{1}{11}\)  
d \(2\frac{7}{9}\)  
8 base = \(\sqrt{30 - 12} = \sqrt{18} = 3\sqrt{2}\)
area = \frac{1}{2} \times 3\sqrt{2} \times \sqrt{12} = \frac{1}{2} \times 3\sqrt{2} \times 2\sqrt{3} = 3\sqrt{6} = \sqrt{54} \text{ cm}^2

9 a irrational
b rational
c rational
d irrational
Unit 2 Answers

2.1 Sequences

1 a arithmetic
   b quadratic
   c arithmetic

2 a 1, 4, 9, 16; 100
   b 2, 8, 18, 32; 200
   c −3, −12, −27, −48; −300
   d 5, 20, 45, 80; 500

3 5th term = 2 × 5^2 + 3 = 53
   10th term = 2 × 10^2 + 3 = 203
   2 × (2 × 5^2 + 3) ≠ 2 × 10^2 + 3

4 T(n) = n^2 + 3

5 a T(n) = n^2 + 7
   b T(n) = 2n^2 + 1
   c T(n) = 3n^2 − 5

2.2 Expanding

1 b y^2 + 9y + 14
   c a^2 + 8a + 15
   d p^2 + 10p + 24
   e n^2 + 15n + 56

2 a d^2 − d − 20
   b z^2 − z − 30
   c q^2 + 3q − 10
   d b^2 − 10b + 21
   e f^2 − 12f + 32
   f b^2 − 14b + 45

3 Dan is correct; Emily’s mistakes were −5 × −6 ≠ −30 and −6x − 5x ≠ x.

4 a 4x^2 − 3x − 35
   b x^2 − 13x − 14

5 LHS = x^2 − 10x + 9x + 9 = x^2 − x + 9
   RHS = x^2 − 7x + 12 + 6x − 3 = x^2 − x + 9
   LHS = RHS

6 a 2x^2 − x − 15
   b 6x^2 − 23x + 20
   c 10x^2 − 13x − 3

7 a 4x^2 + 4x + 1
   b 9x^2 + 12x + 4
   c 25x^2 − 30x + 9
8 a 4\(x^2 \) – 9  
  b 9\(x^2 \) – 1  
  c 25\(x^2 \) – 4  
  d \(x^2 \) – \(y^2 \)

2.3 Factorising

1 b \((x + 5)(x + 3)\)  
  c \((x + 11)(x + 2)\)  
  d \((x - 7)(x - 1)\)  
  e \((x - 4)(x - 6)\)  
2 a \((x + 5)(x - 2)\)  
  b \((x + 4)(x - 3)\)  
  c \((x + 9)(x - 2)\)  
  d \((x + 3)(x - 8)\)  
  e \((x + 2)(x - 10)\)  
  f \((x + 7)(x - 4)\)
3 a \((x + 5)^2\)  
  b \((x + 4)^2\)  
  c \((x - 9)^2\)  
  d \((x - 7)^2\)

2.4 Solving quadratic equations

1 b \(x = -2\) or \(x = 12\)  
  c \(x = 4\) or \(x = 9\)  
  d \(x = -6\) or \(x = 7\)  
2 a \(x = -6\)  
  b \(x = -9\)  
  c \(x = 4\)  
3 Chris is 13 and Jess is 18.  
4 3 or 12  
5 a \(x = -4\) or \(x = 4\)  
  b \(x = -8\) or \(x = 8\)  
  c \(x = -10\) or \(x = 10\)
2 Strengthen

Sequences
1 a quadratic
   b not quadratic
   c not quadratic
   d quadratic
2 a 1, 4, 9, 16, 25; 100
   b –4, –1, 4, 11, 20; 95
   c 7, 16, 31, 52, 79; 304
3 a \( T(n) = n^2 + 2 \)
   b \( T(n) = n^2 + 10 \)
   c \( T(n) = n^2 - 6 \)
4 a \( T(n) = n^2 + 3n \)
   b \( T(n) = n^2 + n + 2 \)
   c \( T(n) = 2n^2 + 5n \)

Expanding
5 a \( x^2 + 10x + 24 \)
   b \( x^2 + 10x + 21 \)
   c \( x^2 + 6x + 5 \)
   d \( x^2 + 5x - 24 \)
   e \( x^2 - 14x + 45 \)
   f \( x^2 - 8x + 12 \)
6 a \( x^2 + 12x + 36 \)
   b \( x^2 - 16x + 64 \)
   c \( x^2 + 2x + 1 \)
7 a \( x^2 - 36 \)
   b \( x^2 - 64 \)
   c \( x^2 - y^2 \)
8 a \( 2x^2 - 5x - 12 \)
   b \( 12x^2 + 17x - 5 \)
   c \( 2x^2 - 15x + 7 \)

Factorising
9 a i 5 and 7
   ii 2 and 6
   b i \((x + 5)(x + 7)\)
   ii \((x + 2)(x + 6)\)
   iii \((x + 4)(x + 5)\)
iv \( (x + 3)(x + 10) \)
v \( (x + 1)(x + 9) \)

10  

a \( -2 \) and \( -5 \)  
b i \( (x - 2)(x - 5) \)  
ii \( (x - 2)(x - 8) \)  
iii \( (x - 5)(x - 6) \)

11 a \( (x - 10)(x + 2) \)  
b \( (x - 7)(x + 4) \)  
c \( (x - 11)(x + 3) \)  
d \( (x - 3)(x + 6) \)  
e \( (x - 4)(x + 6) \)  
f \( (x - 6)(x + 7) \)  
g \( (x - 3)(x + 24) \)

**Solving quadratic equations**

12 b \( x = -8 \) or \( x = -3 \)  
c \( x = -6 \) or \( x = 2 \)  
d \( x = -2 \) or \( x = 14 \)  
e \( x = -9 \) or \( x = -5 \)  
f \( x = -2 \) or \( x = 5 \)  
g \( x = -5 \)  
h \( x = -6 \) or \( x = 8 \)

13 length = 12 cm, width = 7 cm

14 a Alfie is 16.  
b Ben is 13.

2 **Extend**

1 a 3, 10, 19, 30; 2698; 10398  
b 0, 9, 22, 39; 5145; 20295  
c \(-1, -8, -23, -46; -9752; -39502\)

2 a \( T(n) = 2n^2 + 3n - 4 \)  
b \( T(n) = 3n^2 + 5n - 2 \)  
c \( T(n) = -2n^2 - 4n + 5 \)

3 about 33225 comments

4 b 6  
c \( 3(2x - 1) \)

5 b \( x^3 + 5x^2 - 2x - 24 \)  
c \( x^3 - 7x^2 + 2x + 40 \)

6 a \( (2x - 1)(x - 3) \)  
b \( (3x - 2)(x + 2) \)  
c \( (3x + 5)(x - 1) \)
d \(( -2x + 3)(x + 4)\)
e \(( -5x + 2)(x - 3)\)
f \((5x + 1)(-x + 5)\)

7 a \((x - 1)(4x + 3)\)
b \((2x + 1)(2x - 3)\)
c \((3x - 2)(3x + 2)\)
d \((3x + 5)(3x - 1)\)
e \((6x + 1)(2x - 3)\)
f \((4x + 3)(3x - 2)\)

8 length = \(3x + 2\), width = \(2x - 5\), where \(x > 2 \frac{1}{2}\)

9 a \(x = -2 \frac{1}{2}\) or \(x = 1 \frac{1}{2}\)
b \(x = -2 \frac{1}{2}\) or \(x = -\frac{2}{3}\)
c \(x = -1 \frac{1}{2}\) or \(x = \frac{2}{5}\)

10 6 or -4

11 a \(x = -1\) or \(x = -7\)
b \(x = -1\) or \(x = -11\)
c \(x = -1\) or \(x = -21\)

2 Unit test

1 a \(x^2 + 6x + 5\)
b \(x^2 + 2xy + y^2\)
c \(x^2 + 2x - 24\)
d \(x^2 - y^2\)

2 a \(T(n) = 10n^2\)
b \(T(n) = 4n^2\)
c \(T(n) = -3n^2\)

3 a \(x^2 + 14x + 49\)
b \(x^2 + 2xy + y^2\)
c \(x^2 - 6x + 9\)
d \(x^2 - 2xy + y^2\)

4 a \(T(n) = 2n^2 + 7\)
b \(T(n) = 2n^2 + 7n\)
c \(T(n) = 3n^2 - 2n - 4\)

5 a \(2x^2 + 7x - 30\)
b \(3x^2 - 13x + 14\)
c \(12x^2 + x - 6\)

6 a \((x - 5)(x + 8)\)
b \((x - 3)(x + 4)\)
c \((x - 7)(x + 3)\)
7 a \((x - 8)(x + 8)\)
   b \((x - y)(x + y)\)
   c \((5x - 11)(5x + 11)\)
   d \((x + 3)^2\)

8 a \(x = -7\) or \(x = 9\)
   b \(x = -6\) or \(x = -3\)
   c \(x = -10\) or \(x = 1\)

9 \(2n^2 + 10 = 350\)
   \(2n^2 = 340\)
   \(n^2 = 170\)

170 is not a square number so 350 is not in the sequence \(T(n) = 2n^2 + 10\)
Unit 3 Answers

3.1 Inequalities, equations and formulae

1 a ii

\[ x \leq 4 \]

b ii \( x > -6 \)

iii \( y \leq 5 \)

2 a \( x \leq 4 \)

b Yes

3 a \( -4 \leq x < 3 \)

b \( -3 < y \leq 4 \)

c \( 4 \geq z > -6 \)

4 a \( x < -4 \)

b \( x \geq 7 \)

c \( 6 > x > -14 \)

3.2 Using index laws

1 4, 2, 1, \( \frac{1}{2} \), \( \frac{1}{4} \), \( \frac{1}{8} \), \( \frac{1}{16} \)

2 a 1

b 1

c 1

d 1

e 2

3 a 12

b 36

c 6

d 7

4 a \( x^{-5} = \frac{1}{x^5} \)
3.3 Solving equations
1 a \( x = 4 \)
   b \( x = 6 \)
2 a equation
   b formula
   c function
   d formula
   e equation
3 b \( x = -1 \)
   c \( x = -7 \)
4 \( P = 6 \) units

3.4 Changing the subject
1 a \( x = p - 7 \)
   b \( x = m + 10 \)
   c \( x = \frac{y}{k} \)
   d \( x = 3h \)
2 a \( x = \frac{y - 5}{3} \)
   b \( x = \frac{z + 9}{6} \)
   c \( x = \frac{p - 3y}{4} \)
3 a \( r = \frac{A}{2l} \)
3.5 Algebraic fractions

1. \( b \) \( \frac{3y + 7x}{xy} \)
   \( c \) \( \frac{sv - 2w}{vw} \)

2. \( b \) \( x = \frac{1}{3 - y} \)
   \( c \) \( x = \frac{1}{t - 11} \)
   \( d \) \( x = \frac{4}{p + q} \)
   \( e \) \( x = \frac{10}{3n - m} \)

3. \( a \) \( x = \frac{ab}{a + b} \)
   \( b \) \( x = \frac{7mn}{5m - 6n} \)
   \( c \) \( x = \frac{5qv}{qw + 5p} \)

4. \( a \) \( V = \frac{m}{p} \)
   \( b \) \( V = 750 \)
3 Strengthen

Inequalities
1 a diagram iv
   b diagram iii
   c diagram ii
   d diagram i
2 b \(-3 < x \leq 4\)
   c \(8 > x > -2\)
   d \(-1 \leq x \leq 6\)
   e \(-2 \leq x < 1\)

3 a \(x \geq 5\)
   b \(a \leq 5\)
   c \(-2 < m < 5\)

Indices and fractions
4 5
5 a i \(\frac{1}{x^5}\)
   ii \(x^{-5}\)
   b i \(\frac{1}{p^7}\)
   ii \(p^{-7}\)
   c i \(\frac{1}{q^4}\)
   ii \(q^{-4}\)
6 a \(\frac{6a}{8}\)
   b \(\frac{6p}{3q}\)
   c \(\frac{5mp}{np}\)
d \frac{5y}{5r^2v}

7 a \frac{c + d}{cd}

b \frac{2a + 5m}{mn}

c \frac{5wz + xy}{5xz}

Equations and formulae

8 a \quad 11 - x = 4(11 - 3x)

b \quad x = 3

c \quad 8 \text{ units}

9 b \quad x = \frac{y}{8}

c \quad x = \frac{v + t}{t}

d \quad x = \sqrt{\frac{d}{5}}

e \quad x = \sqrt{\frac{m}{n}}

10 a \quad x = \frac{n}{4 - m}

b \quad x = \frac{c - b}{a - 1}

c \quad x = \frac{5}{p + q}

d \quad x = 9.4^2

e \quad x = \frac{25 \times e^2}{6}

11 a \quad 9

b \quad 4x + 7

c \quad 3x + 12

d \quad x = 5

12 a \quad x = 8

b \quad x = 11

c \quad x = 7

13 a \quad x = \frac{1}{y - 10}

b \quad x = \frac{tu}{u + t}

c \quad x = \frac{ab}{5a + 3b}

d \quad x = \frac{7vy}{3y + 2v}
3 Extend

1 a \( \frac{7}{2x^4} \)
   b \( \frac{5}{2x^7} \)
   c \( a^{-3} \)
   d \( r^{-1} \)
   e \( \frac{5}{36^2} \)

2 a \( x \leq 2 \frac{3}{5} \)
   b \( x > -3 \frac{5}{6} \)
   c \( x < 7 \frac{1}{3} \)

3 \( n = -1 \)
4 \( x = 1 \frac{1}{6} \)

5 a \( y = -4x + \frac{5}{2} \)
   b \( y = 2x + \frac{7}{6} \)
   c \( y = \frac{3}{8}x + \frac{5}{8} \)

6 a \( s = \frac{y^2 - u^2}{2u} \)
   b \( s = 40 \)

7 a \( x = -a - 18 \frac{3}{3} \)
   b \( x = \frac{8 - 4y}{3p + \frac{7}{7}} \)
   c \( x = \frac{-3(12q + 5)}{5 - 24q} \)

8 a \( \frac{21y + 4x}{28xy} \)
   b \( \frac{5py + 3qx}{15xy} \)
   c \( \frac{4tu - 6sv}{8st} \)
   d \( \frac{16ac - 15ab}{40hc} \)
9 \( a \) \( x = \frac{1}{a^2 + 8} \)

\( b \) \( x = \sqrt{\frac{uv}{y-u}} \)

\( c \) \( x = \frac{gr(5+p)}{r-q} \)

10 shaded area = area of square – area of circle

\[ A = x^2 - \pi r^2 \]

\[ A = x^2 - \pi \left(\frac{x}{2}\right)^2 \]

\[ A = x^2 - \pi \frac{x^2}{4} \]

\[ A = x^2 \left(1 - \frac{x}{4}\right) \]

3 Unit test

1 54

2 \( x^{-5} \)

3 \( a \) \( x > 2 \)

\( b \) \( x \geq -4 \)

\( c \) \( -3 \leq x < 5 \)

\( d \) \( x \geq 6 \)

4 \( x = 7 \)

5 \( a \) \( x = \frac{x^2-2}{7} \)

\( b \) \( x = \frac{\sqrt{A}}{\sqrt{t}} \)

\( c \) \( x = \frac{7}{a-b} \)

\( d \) \( x = \frac{(st)^2}{5} \)

6 \( a \) \( \frac{4y+5x}{xy} \)

\( b \) \( \frac{aq-6p}{pq} \)

\( c \) \( \frac{st-12u}{3tu} \)
\[ d = \frac{7mp + 4n^2}{7np} \]

7. (a) \[ x = \frac{1}{y + z} \]

(b) \[ x = \frac{tu}{5u + t} \]

(c) \[ x = \frac{2cd}{\sqrt{4c - 5d}} \]
Unit 4 Answers

4.1 STEM: Data collection

1 a secondary data
   b i primary data
      ii questionnaire
   c i primary data
      ii survey

2 a 80 000
   b 200

3 a Not all options are given – what about students who walk?
   e.g. What methods of transport have you used to travel to school this month?
   Car __ Bus __ Train __ Cycle __ Walk __ Other
   b The question is too vague – there are no tick box options so students may just say ‘go home’.
   e.g. How do you fill your time between finishing school and going to bed?
   Do homework __ Play sport __ Watch TV __ Play games __ Other

4 a everyone living in London
   b No; e.g. not everyone in London has a chance of being chosen if the sample is just staff from a food chain.

4.2 Presenting and comparing data

1 a
   1 9
   2 4
   3 2 7 9
   4 2 5 6 7
   5 3
   Key: 1|9 means 19 mm
   b i 7 limpets
      ii 20%
   c i 34 mm
      ii 40.5 mm
2 a

The modal class is 35 to 45 year olds for both male and female members of the club.

3 a The modal class is 35 to 45 year olds for both male and female members of the club.

b There are more male than female cyclists in the group.

4.3 Estimating statistics

1 a 40 cm

b 161.88 cm

2 a 3 hours

b 3 hours and 22 minutes

c 100 people

d 50th item

e 3 to 3.5 hours

4.4 Box plots

1 a iii 32

iv 6

b i 66

ii 61

iii 71

iv 10

2 a i 13°C

ii 11°C

iii 17°C

iv 14°C

v 6°C

b 17°C
b e.g. On average and overall, the temperatures in March are lower in Edinburgh than they are in London.

4.5 Cumulative frequency graphs

1

2 a

b i about 45 years
   ii about 34 years
   iii about 56 years

c i about 80 years
   ii about 22 years
4.6 Histograms

1

![Histogram of 100 m times](image)

2 a 1400 oak trees
b 1770 oak trees

4 Strengthen

Collecting data

1 a 810
b 17 140
c 255 340

2 a primary data
b secondary data
c primary data

3 a e.g. People leaving a swimming pool are likely to be people who take more exercise than the general population.
b e.g. People on a football team are likely to be people who take more exercise than the general population.
c e.g. Students at a school are younger than the average age.
Representing and interpreting data

4 a
0 6 8 9
1 1 3 4 5 5 7 9
2 1 3
b 0|6 means 6

5

6 a 9000 m
b 4000 m
c 5000 m
d 19 mountains
e 4500, 5500, 6500, 7500, 8500
f 36 000, 22 000, 19 500, 15 000, 17 000
g 5763 m (to the nearest m)

7 a 19 values
b 10th item
c 5000 m < H ≤ 6000 m

8 a i 49 cars
   ii 41 cars
   iii 62 cars
b

9 a 25, 47, 60, 69, 74
b
c  i  about 6 crimes  
   ii about 3 crimes  
   iii about 10 crimes  

d  i  0, 20, 20  
   ii about 7 crimes  

10 a  Class width: 80  
      Frequency density: 1, 0.6, 0.4, 0.2, 0.05

b

4 Extend

1 a  Maths
    |  Science
    1 | 9  
   8 7 2 | 1 3 9
   8 7 6 2 | 4 5 8 8 9
8 6 6 5 4 1 1 0 | 4 0 1 3 4 7 8
   8 5 3 1 0 5 | 0 1 2 5
   3 6 | 0

b  i  85%  
   ii 75%  

c  e.g. The two sets of scores are listed separately, and are not identified with particular students, so the 5 who didn’t pass the science test may have all passed the maths test.

2 a  In order: 4, 5, 6, 7, 8, any number from 9 to 12 inclusive, 13, 14, 15
   b  e.g. 7, 7, 7, 9, 11, 14, 15 or 7, 7, 8, 9, 11, 13, 15 or 7,7,7,9,10,15,15

3 a  102.4 (to 1 d.p.)
   b  100–119

4 a  i  47, 48, 51, 33, 56
   ii  35, 24, 49, 53, 62
   iii 71, 144, 83, 116, 95
   b  e.g. The most rain that fell in a month was in 1960. On average the most rain fell in 2012 and the least rain fell in 1999. The most consistent year was 1948 as it had the smallest range. 1960 had the smallest interquartile range so half of the months were more consistent in rainfall.
5 a

![Histogram of 5000 m heats times in 2012](image1)

b

![Histogram of 5000 m heats times in 1992](image2)

c The times for the 5000 m heats were faster in 2012 than in 1992.

4 Unit test

1 250

2 a

```
0| 9
1| 6 9
2| 0 1 3 3 4 5 7 8 9
3| 0 0 3 4 5 5 8
```

Key: 0|9 means 9

b 23

c 9 students

3 a i 150

ii 660 (to the nearest whole number)

b 660 < S ≤ 670
4 a

b about 46 minutes

c about 27 minutes

d about 21 patients
Unit 5 Answers

5.1 Direct proportion

1 Dylan

2 a

\[ y = 8x \]

b Yes

c The answer is 8 for all three pairs of values.

d

e \[ y = 8x \]
3 a No, because the answer to $F + C$ is not the same for each city.

b

![Graph showing temperature in °F and °C]

4 $a = 325.6$ inches, $b = 831$ cm, $c = 253.2$ inches

5.2 Solving problems using direct proportion
1 a $C = 20p$
   b $C = 3b$
   c $S = 12t$
   d $m = 4.5h$
2 a $P = 1.22a$
   b £48.80
3 a $A = 1.8P$
   b 270 Australian dollars
   c £277.78

5.3 Non-linear proportion
1 107.5 m
2 a 40
   b Yes, because $xy = 40$ for each pair of values.
3 a 25
   b 75
4 a 9.6 cm
   b 4.8 cm
5.4 Arcs and sectors of circles

1 a i 56.5 cm²
   ii 30.8 cm

b i 28.3 cm²
   ii 21.4 cm

2 a \(\frac{3}{8}\)

b \(\frac{3}{8}\)

c i 25.1 cm
   ii 9.4 cm

d i 50.3 cm²
   ii 18.8 cm²

3 a i 139.6 cm²
   ii 27.9 cm

b i 67.6 cm²
   ii 27.1 cm

4 28.3 m²

5 Strengthen

Direct proportion

1 a \(C = 74m\)

b \(d = 26h\)

2 a £2.84

b £14.20

c £24.14

3 a = 27, b = 32

4 Yes

5 a 97 rupees

b \(R = 97P\)

c 12 610 rupees

Proportion equations

6 a \(P \propto Q, P = kQ\)

b \(23.4 = 6.5k\)

b \(k = 3.6\)

b \(P = 3.6Q\)

b 54

7 a \(a = 0.36b^2\)

b 43.56

8 a \(s = \frac{45}{t}\)
\[ b \quad 4.5 \]
\[ c \quad 90 \]

**Arks and sectors**

9 a \[ \frac{1}{6} \]

b \[ \frac{1}{8} \]

c \[ \frac{8}{9} \]

10a 5.2 cm

b 177.2 mm

c 30.4 cm

11a 78.5 cm\(^2\)

b 50.2 cm\(^2\)

5 **Extend**

1 \[ p = 2 \text{ cm}, \; q = 13.5 \text{ cm} \]

2 a neither

b inversely proportional

c directly proportional

3 a i \[ \frac{125}{6} \pi \text{ cm}^2 \]

ii \[ \frac{25}{3} \pi \text{ cm} \]

b i \[ \frac{64}{9} \pi \text{ cm}^2 \]

ii \[ \frac{16}{9} \pi \text{ cm} \]

4 a 38.2 mm

b 3.1 cm

c 52.9 mm

5 a 200\(^\circ\)

b 12.7 mm

6 a 42.8 cm\(^2\)

b isosceles

c 24.13 cm\(^2\)

d 18.6 cm\(^2\)

7 169 cm\(^2\)

8 a 20.48

b \( \pm 30.6 \)

9 a 0.03

b £1080

c £60
5 Unit test

1  a  i  Yes
   ii  $C = 62b$
   b  i  No
      ii  $P = 0.05m + 12.95$
2  $x = 19, y = 135$
3  a  $c = 1.375d$
    b  82.5
    c  24
4  No, values of $xy$ are not constant (60, 59.4, 60 and 52.8).
5  a  7.9 cm
    b  23.6 cm$^2$
6  a  1.4
    b  5.425
Unit 6 Answers

6.1 Graphs of quadratic functions

1 a

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

b

\[ y = x^2 \]


c

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td>14</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>9</td>
<td>14</td>
</tr>
</tbody>
</table>

d

\[ y = x^2 + 5 \]

\[ y = x^2 \]

\[ y = x^2 - 5 \]

e They are the same shape and size.

f The \( y \)-intercepts are different.

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KS3 Maths PW Progress Delta 3

2 a

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>48</td>
<td>27</td>
<td>12</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>12</td>
<td>27</td>
<td>48</td>
</tr>
</tbody>
</table>

b

They are all a form of ‘U’ shape

The steepness of the graphs is different.

3 a i  graph C
  ii  graph A
  iii graph E
  iv graph D
  v  graph F
  vi graph B

b

A (0, 0) minimum
B (0, 5) maximum
C (0, 0) minimum
D (0, 0) maximum
E (0, 5) maximum
F (0, 0) maximum

6.2 Solving quadratic equations

1 a
b \( x = -2.6 \) and 2.6  
c \( x = -2.6 \) and 2.6  
d \( x = -2.8 \) and 2.8

2 a

\[ y = 2x^2 \]

\[ x = -1.6 \] and 1.6

3 a

\[ y = x^2 - 5 \]

\[ x = -2.6 \] and 2.6  
c \( x^2 - 5 = 0 \)

4 a

\[ y = (x - 2)(x - 3) \]

\[ x = 2 \] and 3  
c \( (x - 2)(x - 3) \)
5 a $(x - 2)(x - 6)$
   b $x = 2$ and $6$
   c 2 and 6
   d $x - 2 = 0$ when $x = 2$ and $x - 6 = 0$ when $x = 6$

### 6.3 Graphs of cubic functions

1 a

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-27$</td>
<td>$-8$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$1$</td>
<td>$8$</td>
<td>$27$</td>
</tr>
</tbody>
</table>

b

![Graph of $y = x^3$](image)

c rotational symmetry of order 2, centre the origin
They are the same shape and have the same rotational symmetry.

The steepness of the graphs is different.
3

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>27</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-8</td>
<td>-27</td>
</tr>
</tbody>
</table>

b

c They have the same steepness.

d For $y = -x^3$, the $y$-coordinates are positive when $x$ is negative and vice versa.

For $y = x^3$, the $y$-coordinates are negative when $x$ is negative and vice versa.

e a reflection in the $y$-axis (or $x$-axis)

4

a i (2, -8)

ii (-2, 8)

b i $x = 3.3$

ii $x = -2.8, 0, 2.8$

c $x \geq 3.3$
6.4 STEM: Graphs of reciprocal functions

1  

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$-0.5$</th>
<th>$0.25$</th>
<th>$0.25$</th>
<th>$0.5$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-0.33$</td>
<td>$-0.5$</td>
<td>$-1$</td>
<td>$-2$</td>
<td>$4$</td>
<td>$2$</td>
<td>$1$</td>
<td>$0.5$</td>
<td>$0.33$</td>
<td></td>
</tr>
</tbody>
</table>

b

![Graph of a reciprocal function]

c a rotation of 180° about the origin

d i 0.8

   ii $-0.4$

2  

![Graph of a reciprocal function]

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3 a

<table>
<thead>
<tr>
<th>Time, ( t ) (days)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count rate, ( C ) (counts per second)</td>
<td>160</td>
<td>80</td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

b

![Graph showing count rate vs. time](image)

3 c
1. About 56 counts per second
2. About 1.5 days
d No, because it doesn’t start or finish on the same coordinates.
6 Strengthen

Quadratic graphs

1. a

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>$y = 2x^2$</td>
<td>18</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
<td>18</td>
</tr>
</tbody>
</table>

b. $(-3, 18), (-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8), (3, 18)$

c.

![Graph of $y = 2x^2$]

d. $(0, 0)$

e. minimum
2 a

<table>
<thead>
<tr>
<th>x</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x^2 )</td>
<td>18</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>( y = −2x^2 )</td>
<td>−18</td>
<td>−8</td>
<td>−2</td>
<td>0</td>
<td>−2</td>
<td>−8</td>
<td>−18</td>
</tr>
</tbody>
</table>

b

![Graph showing \( y = 2x^2 \) and \( y = −2x^2 \).]

c (0, 0)

d a reflection in the x-axis
3 a

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x²</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>y = x² + 8</td>
<td>17</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>17</td>
</tr>
</tbody>
</table>

b

![Graph of y = x² + 8](image)

c i (0, 8)

ii When x = 0, y = 8

d i (0, 3)

ii (0, 10)

e

![Graphs of y = x² + 3, y = x² + 8, y = x² + 10](image)

f Translate y = x² + 3 up 7 units.

g Translate y = x² + 6 up 14 units.

4 i graph C

ii graph D

iii graph B

iv graph A
5 a

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^2</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>y = x^2 + x - 6</td>
<td>0</td>
<td>-4</td>
<td>-6</td>
<td>-6</td>
<td>-4</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

b

![Graph of y = x^2 + x - 6](image)

c $x = -3$ and $x = 2$

d $(x + 3)(x - 2)$

e The $x$-values in part c are the solutions to $(x + 3)(x - 2) = 0$. 
Cubic graphs

6  a

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^3</td>
<td>-27</td>
<td>-8</td>
<td>-1</td>
<td>0</td>
<td>8</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>-54</td>
<td>-16</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>16</td>
<td>54</td>
</tr>
</tbody>
</table>

b (–3, –54), (–2, –16), (–1, –2), (0, 0), (1, 2), (2, 16), (3, 54)
c

d i about 31
   ii 31.25
   iii Use graph paper and a bigger scale, or draw the graph using ICT.
e i Horizontal line drawn on the graph at y = 40
   ii about 2.7
   iii Yes
f i about 1.7
   ii about –2.7
7 graphs A (a quadratic graph) and C (a reciprocal graph)
Graphs of reciprocal functions

8 a

<table>
<thead>
<tr>
<th>x</th>
<th>-5</th>
<th>-2</th>
<th>-1</th>
<th>-0.5</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-0.4</td>
<td>-1</td>
<td>-2</td>
<td>-4</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

b

![Graph of reciprocal function](image)

c about 0.7

9 a 5 mg/ml

b about 1.3 hours

c 20 mg/ml

d Students check own answers using the formula.

6 Extend

1 a

![Graph of quadratic function](image)

b i about 21 cm$^2$

ii about 2.3 cm

2 i graph C

ii graph D

iii graph B

iv graph A
3  a  \( A = w(15 - w) \)

b

\[ \begin{array}{c}
\includegraphics[width=0.5\textwidth]{image}
\end{array} \]

c  i  56.25 cm\(^2\)
   ii  about 3.5 cm by 11.5 cm

4  a  \((x + 2)(x - 2)\)

b, c

\[ \begin{array}{c}
\includegraphics[width=0.5\textwidth]{image}
\end{array} \]
5 a, bi and bii

iii \( y \) gets bigger and closer to infinity.

c i copy of diagram from Q with smooth hyperbola \( \frac{1}{x+3} \) and dotted line \( x = -3 \) drawn on it; label y-intercept of hyperbola ‘1/3’ (case fraction)

ii copy of diagram from Q with smooth hyperbola \( \frac{1}{2-x} \) and dotted line \( x = 2 \) drawn on it; label y-intercept of hyperbola ‘1/2’ (case fraction)

6 i graph D
ii graph C
iii graph A
iv graph B
6 Unit test

1 a

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>5</td>
<td>-1</td>
<td>-3</td>
<td>-1</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

b

2 a  a reflection in the y-axis
   b  (0, 6)
   c  maximum
   d  $x = -1.7$ and $1.7$

3 a  $(x + 2)(x - 1)$
   b  Graph C, as the solution to $(x + 2)(x - 1) = 0$ is $x = -2$ and $1$, so the graph crosses the $x$-axis at $-2$ and $1$; the function $y = x^2 + x - 2$ has a positive coefficient for $x^2$, so the graph is a ‘U’ shape.

4 i  graph C
   ii graph E
   iii graph D
   iv graph A
   v graph B
Unit 7 Answers

7.1 Rates of change

1 a 80 mph
   b 24 km/h
   c 6.25 m/s
2 9.54 s
3 a 15 m/s
   b 22.5 m/s
   c 37.5 m/s
4 No
5 a driver R
   b driver P
   c 32 mph

7.2 Density and pressure

1 iron 7.9 g/cm³, copper density 8.9 g/cm³, gold 19.2 g/cm³,
2 a 1.25 m³
   b 3000 kg
   c 0.026 64 kg
3 3.7 N/cm²
4 4 704 000 000 N
5 a Yes
   b No

7.3 Upper and lower bounds

1 b upper bound = 65.5 km, lower bound = 64.5 km
   c upper bound = 290.45 g, lower bound = 290.35 g
2 upper bound = 83 499, lower bound = 82 500
3 a 1.3p
   b 1.1p
4 43.175 s ≤ t < 43.185 s
5 ±1 \( \frac{2}{3} \) m/
6 a upper bound = 195 mm, lower bound = 185 mm
   b upper bound = 2340 mm, lower bound = 2220 mm
   c ±60 mm

7.4 Calculating with bounds

1 a upper bound = 2.75 m, lower bound = 2.65 m
   b upper bound = 1.35 m, lower bound = 1.25 m
c $3.7125 \text{ m}^2$

d $3.3125 \text{ m}^2$

2 greatest possible time = 2.45 hours, least possible time = 2.26 hours

3 a upper bound = 38 000 000 mm$^3$, lower bound = 35 000 000 mm$^3$

b $36 000 000 \pm 1 000 000 \text{ mm}^3$

4 upper bound = 3.61 m, lower bound = 3.48 m

7.5 STEM: Accurate measures in real life

1 e.g. The upper bound for the bolt is 1.155 cm but the lower bound for the hole in the nut is only 1.15 cm, so the bolt may be bigger than the hole it needs to fit in.

2 e.g. The worst-case scenario is 475 miles and 49.5 litres, which would be an average of 9.6 miles per litre. So yes, the driver needs to stop for fuel.

3 e.g. Upper bound for cement = 60 $\times$ 25.05 = 1503 kg

Upper bound for pallet and cement = 1503 + 20.5 = 1523.5 kg

Lower bound for the load capacity of the crane = 1.55 tonnes = 1550 kg

1550 > 1523.5

So yes, the crane can safely lift the loaded pallet.

4 a 99kg

b No – the lower bound is 95 kg

7 Strengthen

Compound measures

1 a 17.5 mph

b 50 mph

2 16 minutes and 40 seconds

3 a $8 \text{ kg/m}^3$

b $19.32 \text{ g/cm}^3$

4 $2.4 \text{ N/cm}^2$

Upper and lower bounds

5 a upper bound = 308.5 m, lower bound = 307.5 m

b upper bound = 54.65 million km, lower bound = 53.55 million km

c upper bound = 2.455 m, lower bound = 2.445 m

6 $210 \pm 5 \text{ mm}$

7 a 2.45 cm, 3.95 cm

b $9.6775 \text{ cm}^2$

Accuracy and problem-solving

8 7.48 m/s

9 a greatest possible value = 12.9, least possible value = 12.7

b greatest possible value = 8.5, least possible value = 8.3

c greatest possible value = 1.8, least possible value = 1.7
10 93 m
11 $105 + 50.5 + 25.5 + 10.5 = 191.5$ kg, which is more than the lower bound of 189.5 kg for the maximum load for the cement mixer.
12 $49.5 \div 7.5 = 6.6$, which is only 6 houses

7 Extend
1 a upper bound = 75 476.8 mm$^2$, lower bound = 66 052.0 mm$^2$
   b upper bound = 73 061.7 mm$^2$, lower bound = 68 349.3 mm$^2$
   c e.g. The radius in part a is 150 mm to the nearest 10 mm, so the diameter would be 300 mm to the nearest 20 mm, but in part b the diameter is 300 mm to the nearest 10 mm.
2 the nearest 100 kg
3 about 35 minutes and 20 seconds
4 a 1.376 kg
   b 13.48 N
   c 0.337 N/cm$^2$
   d 0.843 N/cm$^2$
5 a upper bound = 15.387 m, lower bound = 15.353 m
   b upper bound = 9.74 m$^2$, lower bound = 9.70 m$^2$
6 upper bound = 3.98 cm, lower bound = 3.87 cm
7 68.0625
8 a 1000 g ± 5 g
   b 12 000 g ± 60 g
   c 125 g ± 0.625 g
9 upper bound for $p$ and $s$, lower bound for $q$ and $r$

7 Unit test
1 a 62.5 g
   b 61.5 g
2 $495 \text{ ml} \leq C < 505 \text{ ml}$
3 90 mph
4 54 km/h
5 5.5 g/cm$^3$
6 25.5 cm$^2$
7 2.35 m/s
8 e.g. The worst-case scenario is the least amount of storage space the videos (155 minutes) divided by the greatest time per video clip (5.5 minutes); there would be enough space for $155 \div 5.5 = 28.18$ video clips, so she can definitely fit 28 video clips onto one memory stick.
Unit 8 Answers

8.1 Simultaneous equations
1 a $x = 5.5, y = 11$
   b $x = 8, y = 32$
   c $x = 3.25, y = 6.5$
   d $x = 5.6, y = 16.8$
2 a $x = 5, y = 3$
   b $x = 7, y = 4$
3 a $x = 4, y = 2$
   b $x = 7, y = 2$

8.2 Using $y = mx + c$
1 a

![Graph image]

   b gradient = –3, y-intercept = 9
   c $y = -3x + 9$
   d $y = -3x + 9$
   The answers to parts c and d are the same.
2 a i $y = 5x + 2$
   ii $y = \frac{3}{4}x + \frac{7}{4}$
   iii $y = -6x + 8$
   b $y = -6x + 8$
3 a Yes, y-intercept = 5
   b Yes, y-intercept = –6
4 a $y = 4x – 6$
   b $y = 2x – 10$
8.3 More simultaneous equations

1  a  $x = 7, \ y = 9$
   b  $x = 5, \ y = 8$
   c  $x = 4, \ y = 7$
   d  $x = 11, \ y = 2$
   e  $x = 3, \ y = 10$

2  a  $x = 12, \ y = 5$
   b  $x = 7, \ y = 4$
   c  $x = 6, \ y = 11$

8.4 Graphs and simultaneous equations

1  $x = 3, \ y = 1$

2  a  $3a + 2c = 78, \ 2a + 5c = 96$
   b

   ![Graph](image)

   c  Adult £18, Child £12

3  $x = -1, \ y = 2$ and $x = 2, \ y = -1$

4  a  e.g. $x + y = 5$
   b  e.g. Substituting $y = -x + 5$ into $y = -x^2 + 3$ gives $-x + 5 = -x^2 + 3$; this rearranges to give $-x^2 + x - 2 = 0$, which does not have any solutions.
8.5 Solving inequalities

1. a

b

2. a

b \(0 \leq 0 + 1\)

c Yes

d
3

4 a

b No

c

8 Strengthen

Simultaneous equations

1 a \(x = 1, y = 5\)

b \(x = 8, y = 4\)

c \(x = 2, y = 6\)

2 a \(8x = 56\)

b They are eliminated as \(y + -y = 0\)

c \(x = 7\)
d \ y = 6

3 a \ 2x = 24
b \ They are eliminated as \ y - y = 0
c \ x = 12
d \ y = 7

4 a \ 7 = 3m + c
b \ 11 = 5m + c
c \ m = 2, c = 1
d \ y = 2x + 1

5 a \ x = 2, y = 6
b \ x = 2, y = 6

6 gold 12.4 kg, silver 1 kg

**Graphs**

7 a i \ y = 7x + 2
   ii \ y = 3x + 2
   iii \ y = 4x + 5
b i \ gradient = 7, y-intercept = 2
   ii \ gradient = 3, y-intercept = 2
   iii \ gradient = 4, y-intercept = 5

8 a \ x = 5
b \ (5, 0)
c \ y = -2
d \ (0, -2)

8 e–g

h \ (7.5, 1)
Inequalities

9 a

b

c

10

11

12

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8 Extend
1 a \(x = 2, y = -6\)
   b \(x = -3, y = 12\)
2 \(y = -3.5x - 39.5\)
3 a \(x + y = 14\) is half of \(2x + 2y = 28\)
   b 42
   c \(2x = 10\)
   d \(x = 5, y = 9\)
   e \(x = 3, y = 8\)
4
5 \(x = 6, y = 5\)
6 a one solution, \(x = -3, y = -5\)
   b infinitely many solutions
   c one solution, \(x = 0, y = -5\)
7 cake 75p, biscuit 30p
8 a \(a = 5\) cm, \(b = 8\) cm
   b \(a = 13.4^\circ, b = 7.4^\circ\)

8 Unit test
1 a i gradient = 2
   ii y-intercept = 9
   b i gradient = \(\frac{3}{4}\)
   ii y-intercept = -5
2 \(y = 4x - 16\)
3 a \(x = 10, y = 3\)
   b \(x = 4, y = 7\)
   c \(x = 2, y = 9\)
4
5 a \(3x + y = 79\)
b $7x + y = 151$

c $x = £18, \ y = £25$

6 a $x = 8, \ y = 5$

b $x = 3, \ y = 5$
Unit 9 Answers

9.1 The tangent ratio
1 b

2 a 2.1
   b 0.8

3 a \tan \theta = \frac{3}{4}
   b \tan \theta = \frac{5}{6}
   c \tan \theta = \frac{7}{5}

4 a 4.8 cm
   b 4.2 cm
   c 8.0 cm
   d 3.7 cm
   e 9.0 cm

9.2 The sine ratio
1 a 0.8
   b 0.4
   c 1.0

2 a \sin \theta = \frac{5}{8}
   b \sin \theta = \frac{7}{12}
   c \sin \theta = \frac{9}{15}

3 a 4.5 cm
   b 7.2 cm
   c 12.3 cm
   d 5.9 cm
   e 16 cm
   f 13.9 cm
4 a i sine ratio
   ii 9.2 cm
b i tangent ratio
   ii 10 cm

9.3 The cosine ratio
1 a 0.9
   b 0.1
   c 0.7
   d 1.0
2 a $\cos \theta = \frac{7}{9}$
   b $\cos \theta = \frac{9}{12}$
   c $\cos \theta = \frac{3}{10}$
3 a 4.8 cm
   b 7.7 cm
   c 4.8 cm
   d 21.3 cm
   e 12.4 cm
   f 11.7 cm
4 a i tangent ratio
   ii 4.7 cm
b i cosine ratio
   ii 10.6 cm

9.4 Using trigonometry to find angles
1 a i 24.6°
   ii 36.9°
   b i 39.8°
   ii 54.5°
   c i 38.9°
   ii 72.5°
2 a 50.2°
   b 56.3°
   c 57.0°
   d 47.5°
3 5.7°
4 054°

9.5 Solving problems using trigonometry
1 a 54.0°
b 65.0°  
c 46.2°  
2 a 153.2 km  
b 128.6 km  
3 41.5 cm²  
4 24.2°  
5 27.9°  

9.6 Trigonometric graphs  
1 a

<table>
<thead>
<tr>
<th>θ</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
<th>210°</th>
<th>240°</th>
<th>270°</th>
<th>300°</th>
<th>330°</th>
<th>360°</th>
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<td>0.9</td>
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<td>0.9</td>
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<td>−0.9</td>
<td>−1</td>
<td>−0.9</td>
<td>−0.5</td>
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</tr>
</tbody>
</table>

b

2 a

<table>
<thead>
<tr>
<th>θ</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
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<th>150°</th>
<th>180°</th>
<th>210°</th>
<th>240°</th>
<th>270°</th>
<th>300°</th>
<th>330°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td>cos θ</td>
<td>1</td>
<td>0.9</td>
<td>0.5</td>
<td>0</td>
<td>−0.5</td>
<td>−0.9</td>
<td>−1</td>
<td>−0.9</td>
<td>−0.5</td>
<td>0</td>
<td>0.5</td>
<td>0.9</td>
<td>1</td>
</tr>
</tbody>
</table>

b

c A translation left 90°

3 a 90°, 270°  
b 120°, 240°  
c 0°, 360°
4 a $60^\circ$
  b $\sqrt{3}$
  c $\sqrt{3}/2$
  d $0.8660$

9 Strengthen

**Unknown sides**

1 a

![Right triangle with labels](image)

b

![Right triangle with labels](image)

c

![Right triangle with labels](image)

2 a i $\tan \theta = \frac{12}{16}$
  ii $\sin \theta = \frac{12}{20}$
  iii $\cos \theta = \frac{16}{20}$

b i $\tan \theta = \frac{36}{77}$
  ii $\sin \theta = \frac{36}{85}$
  iii $\cos \theta = \frac{77}{85}$

b i $\tan \theta = \frac{15}{8}$
  ii $\sin \theta = \frac{15}{17}$
  iii $\cos \theta = \frac{8}{17}$

3 a 17.2 cm
  b 7.0 cm
  c 6.8 cm

4 a Students label $x$ cm ‘opp’ and 9 cm ‘hyp’.

b sine ratio

c $\sin 62^\circ = \frac{x}{9}$
d. \( x = 9 \times \sin 62^\circ \)
e. 7.9 cm

5. a. 15.2 cm  
b. 6.0 cm  
c. 8.7 cm

**Unknown angles**

6. a. 44.4°  
b. 14.5°  
7. a. 51.3°  
b. 36.9°  
8. a. 16.7°  
b. 35.0°  
9. a. 32.0°  
b. 36.9°  
c. 54.3°

10. a. Students label 4 cm 'adj' and 9 cm 'hyp'.  
b. cosine ratio  
c. \( \cos \theta = \frac{4}{9} \)  
d. \( \theta = \cos^{-1} \frac{4}{9} \)  
e. 63.6°

11. a. 47.2°  
b. 52.4°  
c. 45.6°

12. | \( \theta \) | 0° | 30° | 90° | 150° | 180° | 210° | 270° | 330° | 360° |
<table>
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<tbody>
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<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>–0.5</td>
<td>–1</td>
<td>–0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

**Solving problems**

13a. sine ratio  
b. 5.7 m  
14. 036°

15. 5.9 cm

**9 Extend**

1. a. 9.4 cm  
b. 13.6 cm  
c. 10.2 cm  

2. a. 52.4°  
b. 51.1°
c 45.4°
3 a 52.0 km
  b 13.9 km
4 a 33.7°
  b 56.4°
  c 89.9°
5 40.4°
6 a 5.35 m
  b 21.4°
7 a 8.9 cm
  b 29.2°
  c 10.25 cm
8 a 21.2 cm
  b 48.5°
  c 16 cm
9 a

9 Unit test
1 a \( \sin \theta = \frac{20}{29} \)
  b \( \cos \theta = \frac{21}{29} \)
  c \( \tan \theta = \frac{20}{21} \)
2 a 14.5 cm
  b 9.3 cm
3 a 57.8°
  b 54.8°
4 1.4 m
5 a 5.8 cm
  b 17.8 cm
  c 51.3 cm²
6 12.3 cm
7 a
b  $30^\circ, 150^\circ$
Unit 10 Answers

10.1 Explain, show and justify

1 He is correct because the first prime number is 2, which is even. The next prime number is 3, which is the smallest odd prime number. 1 is a smaller odd number but it is not prime.

2 Yes, because mean = \( \frac{112}{23} = 4.9 \), and 4.9 is greater than 3.

3 a area of first shape = \((x + 4)(x + 7)\) and area of second shape = \(x^2 + 11x + 28\)
   
   b The length of each rectangle is \(x + 7\) and the width of each rectangle is \(x + 4\).
   
   As both rectangles have the same length and the same width, the areas must be the same.
   
   So the area of the first rectangle, \((x + 4)(x + 7)\), must be equal to the area of the second rectangle, \(x^2 + 11x + 28\).

4 False
   
e.g. 20% of 80 = \((10\% \text{ of } 80) \times 2 = 8 \times 2 = 16\), but \(80 \div 20 = 4\)
   
   So dividing by 20 does not give 20% of a number.

5 a True
   
   RHS = \(4(x + 3) = 4 \times x + 4 \times 3 = 4x + 12\) = LHS

   b False
   
   LHS = \((x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 25 = x^2 + 10x + 25 \neq\) RHS

   c False
   
   LHS = \((x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9 \neq\) RHS

6 a LHS = \((p + q)(p - q) = p^2 - pq + pq - q^2 = p^2 - q^2\) = RHS

   b LHS = \(3(a + 5) - 4(a + 2) = 3a + 15 - 4a - 8 = -a + 7 = 7 - a\) = RHS

7 Wick for each candle: upper bound = 20.5 cm, lower bound = 19.5 cm
   
   Total candle wick: upper bound = 505 cm, lower bound = 495 cm

   Worst-case scenario: \(495 \div 20.5 = 24.15\)
   
   24.15 < 25, so there won't be enough for 25 candles.

10.2 MODELLING: Real-life situations

1 a Students draw a straight line with positive gradient through the crosses on the graph.

   b i about 32 million \(\text{km}^2\)
      
      ii about 35 million \(\text{km}^2\)

   c e.g. The crosses on the graph indicate a curve rather than a straight line, and that the area of the hole in the ozone layer has decreased overall since 2006.
2 a

![Graph of Cost (£) vs Number of prints](image)

b Snap Happy
c 60 photos

3 a ii 73.5 cm$^3$
iii 72 cm$^3$
iv 48 cm$^3$

b

c 5 cm
d The length of the square base of the box = 30 – 2
The height of the box = l
The volume of the box = width \times length \times height
= (30 – 2l) \times (30 – 2l) \times l = (30 – 2l)^2 \times l = l(30 – 2l)^2

4 a e = 2m
b For every 1 kg added, the extension will be 2 cm, so when m = 1, e = 2, when m = 2, e = 4, etc.
c 11.5 kg
d The assumption is that the extension is proportional to the mass until the spring breaks.
10.3 Proof

1 Brad’s statement is a proof and Poppy’s is an example.

2 \( n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6) = 7n + 21 = 7(n + 3) \)

3 Liam’s answer is a proof and Beth’s answer is a demonstration.

4 a True

e.g. The two odd numbers can be written as \(2n + 1\) and \(2m + 1\), where \(n\) and \(m\) are integers.

\((2n + 1)(2m + 1) = 4mn + 2n + 2m + 1 = 2(mn + n + m) + 1\)

\(2(mn + n + m)\) is a multiple of 2 and so is even.

Hence \(2(mn + n + m) + 1\) is odd.

b False, e.g. \(2 + 3 + 4 = 9\)

c False, e.g. any right-angled scalene triangle with all three sides of different length

10.4 More proof

1 1st term = \(n\), 2nd term = \(n + 10\), 3rd term = \(n + 20\), 4th term = \(n + 21\)

\[ L(n) = n + n + 10 + n + 20 + n + 21 = 4n + 51 \]

2 a The answer is always 15.

b \((n \times 2 + 10) \times 3 \div 2 = 3(2n + 10) \div 2 = 3n + 15 – 3n = 15\)

3 Area of the square = \(x^2\), area of the circle = \(\pi \left(\frac{x}{2}\right)^2\)

Area of remaining paper = \(x^2 - \pi \left(\frac{x}{2}\right)^2 = x^2 - \frac{x^2 \pi}{4} = x^2(1 - \frac{\pi}{4})\)

4 \(5n^2 = 5000, n^2 = 1000\)

1000 is not a square number so 5000 is not a term in the sequence with \(n\)th term \(5n^2\).

5 a e.g. \(n = 0.5\)

b e.g. \(\frac{1}{n} < 1\) for any integer

6 \(x^2 + 9x + 20 = (x + 4)(x + 5)\) and \(x + 4\) and \(x + 5\) are consecutive numbers.

7 a e.g. An even number can be written as \(2n\); \(2n + 2 = n, n\) is a whole number.

b e.g. \(5 \times 5 = 5^2\), so \(5x = x^2\) when \(x = 5\). Another exception is \(x = 0\).

c Four consecutive numbers will always be either even, odd, even, odd or odd, even, even, odd, even.

Odd \(\times\) even\(=\) even, and even \(\times\) odd \(=\) even

So the product of all four numbers is even \(x\) even, which is even’

10 Strengthen

Explain, show and justify

1 Georgia has given a mathematical explanation and Callum has given a demonstration.

2 For the terms in a sequence, \(n\) is always an integer so \(2n\) is an even number as it is a multiple of 2. \(2n - 1\) is 1 less than a multiple of 2, so it is an odd number.

3 a \(x^2 + 20x + 100\)

b \(x^2 + 2x + 2x + 4 - 4 = x^2 + 4x\)

c \(LHS = x^2 + 5x - 5x - 25 = x^2 - 25 = RHS\)
d $\text{RHS} = x^2 + x - x - 1 = x^2 - 1 = \text{LHS}$

4 a False

LHS = $x^2 + 7x + 10x + 70 = x^2 + 17x + 70 \neq \text{RHS}$

b True

LHS = $x^2 - 7x + 10x - 70 = x^2 + 3x - 70 = \text{RHS}$

c True

LHS = $(x + 3)^2 - 9 = (x + 3)(x + 3) - 9 = x^2 + 3x + 3x + 9 - 9 = x^2 + 6x = \text{RHS}$

**Modelling**

5 a

![Graph showing distance over time](image)

b 2 hours

c e.g. The graph assumes that he travelled at a constant speed for the whole journey but he probably didn’t.

6 a i Deal A £9, Deal B £15

ii Deal A £21, Deal B £23

iii Deal A £30, Deal B £29

iv Deal A £39, Deal B £35
b

![Graph showing cost vs number of texts](image)

more than 900 text messages

Proof

7  a  i  3
   ii  5
   iii  7
   iv  odd

b  Students test more pairs of consecutive numbers

c  \( n + 1 \)

d  \( n + n + 1 = 2n + 1 \)

e  \( 2n + 1 \) is always odd as \( 2n \) is a multiple of 2, and a multiple of 2 plus 1 is always an odd number.

8  \( n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1) \)
   \( 3(n + 1) \) is always a multiple of 3.

10 Extend

1  a  RHS = \((x + 2)(x + 3) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6 = \text{LHS}\)

b  LHS = \((x + 2)^2 + (x - 2)^2 = (x + 2)(x + 2) + (x - 2)(x - 2)\)
   \(= x^2 + 2x + 2x + 4 + x^2 - 2x - 2x + 4\)
   \(= 2x^2 + 8 = \text{LHS}\)

2  a  LHS = \(\frac{q}{pq} + \frac{p}{pq} = \frac{p + q}{pq} = \text{RHS}\)

b  LHS = \(\frac{2p}{q} + \frac{3q}{p} = \frac{2p^2}{pq} + \frac{3q^2}{pq} = 2\frac{p^2 + 3q^2}{pq} = \text{RHS}\)

c  LHS = \(\frac{p}{q} + \frac{q}{r} + \frac{r}{p} = \frac{p^2r}{pqr} + \frac{pq^2}{pqr} + \frac{qr^2}{pqr} = \frac{p^2r + pq^2 + qr^2}{pqr} = \text{RHS}\)

3  e.g. \(n\text{th term} = 2n^2 + 4n - 1\)
   \((n - 1)\text{th term} = 2(n - 1)^2 + 4(n - 1) - 1\)
   \(= 2(n^2 - 2n + 1) + 4n - 4 - 1\)
   \(= 2n^2 - 4n + 2 + 4n - 5\)
   \(= 2n^2 - 3\)

Consecutive terms in the sequence are odd, since \(2n^2\) must be even.

Odd + odd = even.
4 Using the properties of angles on parallel lines, two of the angles in the triangle are 85º and 48º. Using the angle properties of triangles \(x = 85º + 48º = 133º\).

5 The line \(y = 3 - 2x\) crosses the \(x\)-axis at \((1 \frac{1}{2}, 0)\), so the base of the triangle is \(1 \frac{1}{2}\) units.

The two lines intersect at \((\frac{3}{5}, \frac{1}{5})\), so the height of the triangle is \(\frac{4}{5}\).

Area = \(\frac{1}{2} \times 1 \frac{1}{2} \times \frac{3}{2} \times \frac{9}{5} = \frac{27}{20} = 1 \frac{7}{20}\)

6 Height of the triangle = \(\sqrt{3^2 - \left(\frac{3}{2}\right)^2} = \sqrt{9 - \frac{9}{4}} = \frac{3\sqrt{3}}{2}\)

Area of triangle = \(\frac{1}{2} \times 3 \times \frac{3}{2} \sqrt{3} = \frac{9}{4}\sqrt{3}\)

7 Lower bounds for the dimensions are 6.5 cm, 7.5 cm and 18.5 cm, so the lower bound for the capacity is 901.875 cm³.

1 litre = 1000 cm³, so the carton is not big enough for 1 litre of smoothie.

8 For the terms in a sequence, \(n\) is always an integer.

\(2n + 2 = 2(n + 1)\), which is a multiples of 2 and so is an even number.

9 Upper bound for the height = 20.5 m.

\(\tan 50º = \frac{20.5}{\text{adj}}\)

Maximum horizontal distance = 17.2 m

10 Area of circle = \(\pi r^2\)

Base of the equilateral triangle: \(\sin 60º = \frac{\text{half of base}}{r}\) so base = \(\sqrt{3} r\)

Height of the equilateral triangle = \(r + \sqrt{r^2 - \frac{3}{4}r^2} = \frac{3r}{2}\)

Shaded area = \(\pi r^2 - \frac{1}{2} \times \sqrt{3} r \times \frac{3r}{2} = \pi r^2 - \frac{3\sqrt{3}r^2}{4} = r^2\left(\pi - \frac{3\sqrt{3}}{4}\right)\)

11 a Students draw a reflex angle.

b Students bisect their reflex angle using a pair of compasses (showing their construction marks).

c obtuse angles

d two obtuse angles

e A reflex angle \(r\) is \(180º < r < 360º\).

Half of \(r\) is \(90º < \frac{r}{2} < 180º\), which is an obtuse angle.

12 a 8 has the factors 1, 2, 4 and 8.

A multiple of 8 which is more than 8 has at least the factors 1, 2, 4 and 8 and itself.

Therefore a multiple of 8 has at least four factors.

b 8 has the factors 1, 2, 4 and 8.

A multiple of 8 which is more than 8 has at least the factors 1, 2, 4 and 8 and itself, so a multiple of 8 which is more than 8 will always have at least five factors. Therefore 8 is the only multiple of 8 with exactly four factors.

13 \(a + w = 180º, b + x = 180º, c + y = 180º\) and \(d + z = 180º\) because each pair of angles make a straight line.

\(a + w + b + x + c + y + d + z = 180º + 180º + 180º + 180º = 720º\)

The angles in a quadrilateral total 360º, so subtracting \(a, b, c\) and \(d\) gives \(w + x + y + z = 360º\)
10 Unit test

1 An integer is a multiple of 4; 4n + 2 is a multiple of 4 add 2.
   \[ 4n + 2 = 2(2n + 1) \]
   2(2n + 1) is a multiple of 2 and so is an even number.

2 An obtuse angle is more than 90º and less than 180º, so the sum of two obtuse angles is always
   greater than 180º. As the angles in a triangle total 180º there cannot be more than one obtuse
   angle in a triangle.

3 For \( \gamma = 5x \), when \( x = 1, \gamma = 5 \times 1 = 5 \)
   So the line \( y = 5x \) goes through the point \((1, 5)\), not the point \((1, 4)\).

4 LHS = \((x - 3)^2 + 6x = x^2 - 3x + 9 + 6x = x^2 + 9 = \) RHS

5 a e.g. \(-4 + -2 = 2 \)
   b e.g. \((-2)^3 = -8 \)

6 Area of square = \( r^2 \), area of sector = \[ \frac{\pi r^2}{4} \]
   Shaded area = \( r^2 - \frac{\pi r^2}{4} = r^2 (1 - \frac{\pi}{4}) \)

7 \((2n + 1)(2n + 3) = 4n^2 + 8n + 3 \)
   \[ 4n^2 + 8n = 4(n^2 + 2n) \]
   4\(n^2 + 2n\) is even so 4\((n^2 + 2n)\) + 3 is always odd.

8 Total = \( n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2) \)
   Mean = total ÷ 5 = \( 5(n + 2) ÷ 5 = n + 2 \)
   \( n + 2 \) is the middle number.

9 \( a + d = 180º, b + e = 180º \) and \( c + f = 180º \) because these pairs of angles form straight lines.
   \[ a + d + b + e + c + f = 180º + 180º + 180º = 540º \]
   As the angles in a triangle total 180º, \( d + e + f = 540º - 180º = 360º \)

10 Upper bound for the area = \( 40.5 \) cm\(^2\)
   Lower bound for the length = 7.5 cm
   Maximum width = \( 40.5 ÷ 7.5 = 5.4 \) cm

11 \( 2n^2 + 4n + 10 = 2(n^2 + 2n + 5) \)
   \( 2(n^2 + 2n + 5) \) is a multiple of 2 and so is always even.
   Reece's hypothesis is true.